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Optimum Conditions of Size and Volume of Potatoes in a Boiling Process Based on a Porous Model

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ポーラスモデルをベースにした煮る加熱過程における ジャガイモの最適なサイズと体積の条件

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Summary:

In the present study, by using a mathematical model based on the porous media theory for a cooking process, thermal fluid convection is numerically examined for a food with a surrounding liquid. Potatoes as foodstuff are boiled in a container filled with water by heating from below. Then we numerically predict not only variations of the foodstuff temperature, but also variations of the liquid temperature and velocity. The results are partially compared with experimental data. It is found that there exists a significant flow parameter (Darcy number), which plays an important role on rapid thermal convections and homogeneous temperature distribution over the container. It is also found that there exists a range of this parameter in which the heat transfer is most promoted. Since this parameter determines the relation between average size and volume ratio of the potatoes in the liquid, the optimum size and volume ratio of the potatoes for a homogeneous heating can be predicted.

Key words: cooking, convection, heat transfer, Darcy number, porous media

1　Introduction

It is known in cooking processes that boiling is different from others like deep frying and roasting[1], where Fig. 1 shows a schematic diagram of these typical three types of cooking. Particularly, in the boiling process, foodstuff is not only heated mildly for a long time in a surrounding liquid, but also the liquid and ingredients seep into and out of the foodstuff. Therefore, the boiling process is complicatedly influenced by heating strength, thermal

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properties, shapes of a container and materials, solute concentrations, void fraction and so on. In such a complicated situation, it is important to know the most suitable heating conditions when foods are boiled, where heat and mass transfer plays an important role between foodstuff and surrounding fluids.

In spite of this, most of the investigations which have been made[2-5] are mainly focused on temperature variations inside the foodstuff for a given surrounding liquid temperature and relations between hardness and temperature of the foodstuff.

Since the surrounding liquid temperature determines the variation of the foodstuff temperature and hardness, however, it is important to know spatial and temporal variations of the surrounding liquid temperature in the container when heated from below. From the viewpoint of this, the objective in the present study is to make a theoretical prediction of the temperature of the surrounding liquid as well as the foodstuff. In the present work, we propose a mathematical model based on the porous media theory[6,7], by which not only heat and mass transfer in the foodstuff and surrounding liquid, but also variations of temperature, hardness and solute concentration inside the food materials can be predicted. Such investigations enable us to predict a suitable and precise condition for cooking and to develop a boiling process for an ecological and energy saving cooking.

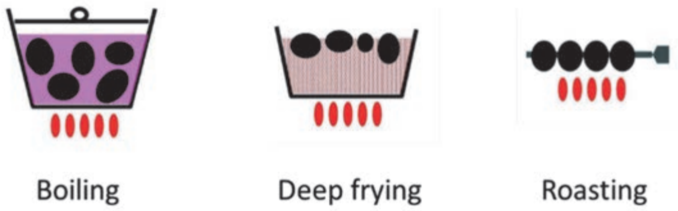


Figure 1 Typical three types of cooking process.

2 Methods

2.1 Experimental setup and measuring system

In the experiments, potatoes in a container filled with water are heated from below by the IH cooker (Panasonic KZ-DS13, 100V, 1200W). The temperature is measured every 5 minutes at several points in the water and potatoes as is shown in Fig.2, where the size of the container is of R=H=0.1m, the heating region is in $0.4R \leq r \leq 0.7R$ when r denotes the radial coordinate and R and H denote the radius and height of the container, respectively.

The average weight of the potatoes is 1.7kg ± 0.05kg and the average sizes of potatoes consist of two groups where the average size of the large group is 3.2cm and that of the small group is 0.65cm in diameter. The liquid

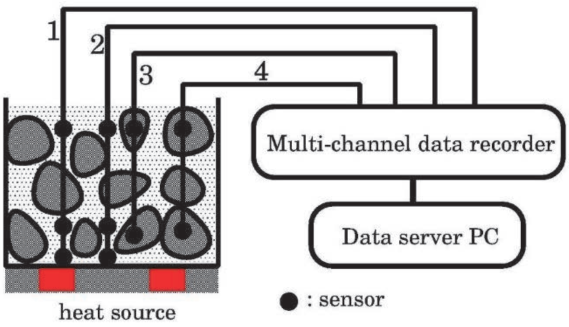


Figure 2 Experimental setup and measuring system.

is of deionized water whose volume is 2 ℓ, so that the void fraction is Ø=0.549. Temperature data are measured by the internal temperature probes (Anritsu) and stored in the PC through the multi-channel data recorder (Anritsu, Thermo logger, AM8000) as in Fig.2.

2.2 Numerical procedure

2.2.1 Three-dimensional model

In order to predict local temperatures and liquid velocities in the container, we assume that configurations and displacements of the foodstuff are like porous media. Based on this model, we numerically examine the thermal and fluid motions in the porous media filled with liquid in the container.

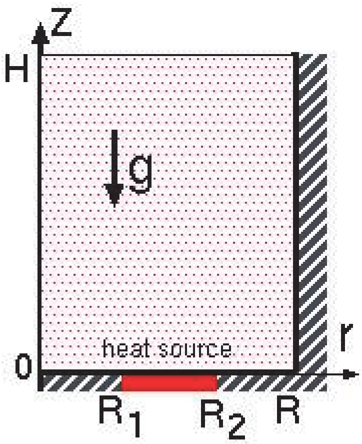


Figure 3 Region of numerical calculations.

In the (r, θ, z) cylindrical coordinate system, the following equations are obtained under the Boussinesq approximation[8-10] for $0 \leq r \leq R/H=1$, $0 \leq z \leq 1$ in the non-dimensional form (see Fig.3):

$$\nabla \cdot \mathbf{v}=0 , \qquad \text{(continuity equation)}$$

$$[\phi^{-1}\partial \mathbf{v}/\partial t+\phi^{-2}(v \cdot \nabla \mathbf{v})] = -\nabla P+ (Pr/\phi)\nabla^2\mathbf{v}- (Pr/Da)\mathbf{v}$$
$$+ PrRa[0, 0, (T-T_0)], \qquad \text{(momentum equation)}$$

$$\partial T/\partial t + \sigma\ v \cdot \nabla T=\nabla^2 T, \qquad \text{(energy equation)}$$

where $\mathbf{v}$= (u, v, w) denotes the fluid velocity vector and T the fluid temperature. The following non-dimensional parameters are introduced:

$Ra=g\beta\ \Delta T\ H^3/(\nu\alpha_m)$ (Rayleigh number), $Da=K/H^2$ (Darcy number), $Pr = \nu/\alpha_m$ (Plandtl number), $K = Dp^2\phi^3/ [180(1-\phi)^2]$ (permeability), ϕ (void fraction) , Dp (effective diameter of the potatoes).

The heat parameters are as follows: $(\rho c)_m=(1-\phi) (\rho c)_s +\phi (\rho c)_f$, $k_m=(1-\phi) k_s+\phi k_f$, $\alpha_m =k_m / (\rho c)_m$, $\sigma = (\rho c)_f/ (\rho c)_s$, where ρ and c denote the density and the specific heat and $\sigma = (\rho c)_f/(\rho c)_m$, while the suffixes s, *f* and m are referred to as the solid, liquid and mixed phase, respectively. On the other hand, the heating area is $\Delta S=0.33S=0.33\pi R^2$, since the heating region is given as 0.4R<r<0.7R as in Fig.3. The characteristic temperature ΔT is given as the input heat flux q_{in} at the bottom through $q_{in}=k_m\Delta T/H$. We note that Da increases when Dp or ϕ increases.

2.2.2 Numerical method and parameters

In the numerical calculations the finite difference method is used, where HSMAC for the pressure terms, Adams-Bashforth and Crank-Nicolson methods for the viscous terms are specifically adopted in the staggered grid mesh[11]. The mesh grid points are adopted to be 48×24×48 in (r, θ, z) directions for $0 \le r \le 1$, $0 \le \theta \le 2\pi$, $0 \le z \le 1$, which is schematically depicted in Fig.4. In most of the calculations the time step Δt is set to be 10^{-6}.

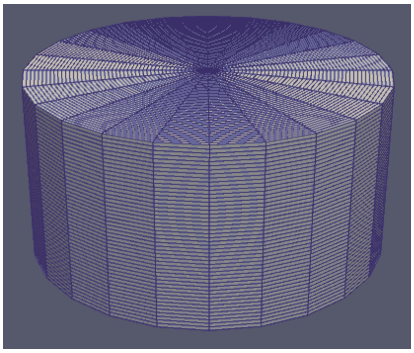


Figure 4 Mesh-grid-configuration used in the present calculations.

The following boundary conditions are considered in the present calculations:

(i) $\partial u/\partial z = \partial v/\partial z=0$, $w = 0$, $\partial T/\partial z =0$ at z=H,

(ii) $u = v = w=0$, $\partial T/\partial r = 0$ at r=R,

(iii) $u = v = w=0$ of 0<r<R at z=0, while (a) $T|_{z=0}-T|_{z=1} =1$ or (b) $\partial T/\partial z\,|_{z=0}=-1$ for $0.4R\le r\le 0.7R$ and otherwise $\partial T/\partial z|_{z=0} = 0$.

The numerical parameters are set to be ϕ=0.549, R/H=1, Pr=6.67 (fixed), while Da and ΔT are variable.

3 Results and Discussions

3.1 Experimental results and comparison with the lumped model

We first show in Fig.5 the comparison of the experimental results with the calculations by the lumped (one-dimensional) model[12-14], where two different sizes of potatoes in water are heated by the IH cooker. It is found in Fig.5 (a) that the experimental results agree rather well with the calculations when the size of the potatoes are small. When the potatoes are large as in Fig.5 (b), the temperature variations at the center of the potatoes (thick broken line) are delayed from that of liquid phase (thick solid line) due to the duration of heat conduction.

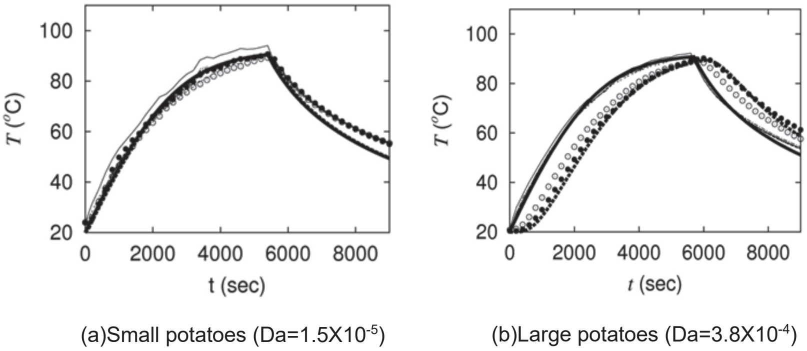


Figure 5 Comparison between the experimental data and calculations bassed on the lumped model, where solid thick lines (liquid temperature) and broken lines (center of the potatoes) are calculated by the lumped model, while ○ denotes temperature in the center of the potatoes at the top of the container, ● at the bottom, and thin lines (liquid temperature) indicate the measurments.

3.2 Numerical results

3.2.1 Onset of convection

When the temperature difference between upper and lower surfaces increases above a critical Ra[15], it is the buoyancy that derives fluid motions. Then, the analytical thermal-fluid convections begin to occur in different patterns of temperature as in Fig6, where the red regions denote the upward flow and higher temperature, while the blue regions denote the downward flow and lower temperature[9]. On the other hand, Fig7show the 3D-numerical results of the convection pattern for different aspect ratio s, corresponding to the above analytical results. It is found that the both patterns of convection qualitatively agree with each other.

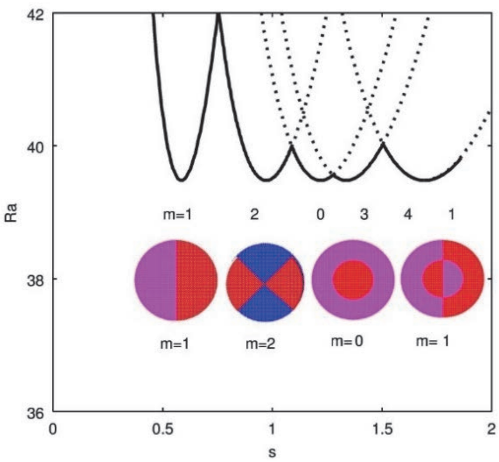


Figure 6 Analytical results of convection patterns for Ra when the aspect ratio s (=R/H) varies, where the blue denotes the lower temperature and the red the higher temperature of the liquid.

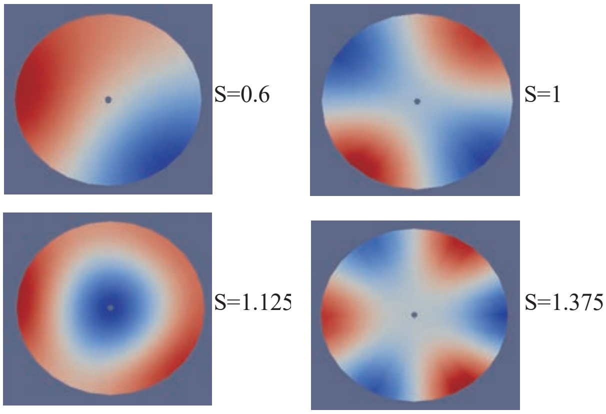
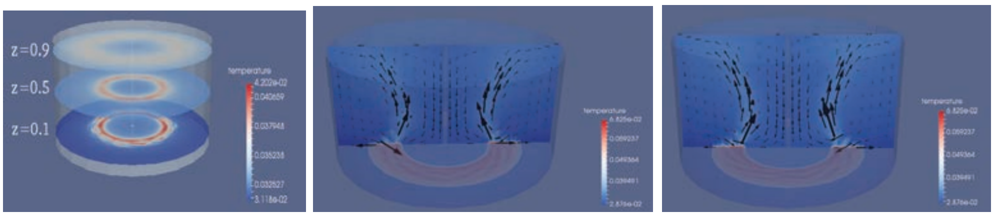


Figure 7 3D-numerical results of convection patterns for different aspect ratio s, where z=0.5 and Ra = 6.73×10^{10}. These results can be compared with the analytical results in Fig.6.

3.2.2 Fully developed convection

Figure 8 shows the 3D numerical results in the temperature and velocity distributions when ΔT=100 (corresponding to $q_{in}\Delta S$= 5.82W) at t=0.1 (corresponding to 1.89hour). From these figures, it is found that the temperature is annular in distribution as in Fig. (a), while the velocity distribution is almost axisymmetric (independent of azimuthal directions as in Fig. (b) and (c)). Resulting from this, we can say that the temperature and velocity distributions are almost axisymmetric. Resulting from this, it is expected that the 2D analysis in the r-z coordinate system is sufficiently available.



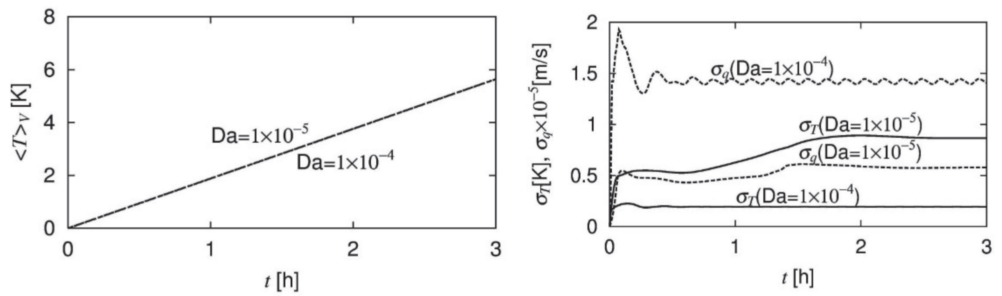
(a) **(b)** **(c)**

Figure 8 Typical results in 3D numerical calculations, where (a) Temperature distribution ($T\times10^{-2}$ [K]), (b) Velocity distribution for θ=0, π (v $\times1.5\times10^{-6}$ [m/s]) and (c) Velocity distribution for $\theta=\pi/2$ and $3\pi/2$.

3.2.3 Smaller heat intensity (2D analysis)

Figure 9 (a) shows that the average temperature increases with time when ΔT=100 ($q_{in}\Delta S$=5.42W) for different

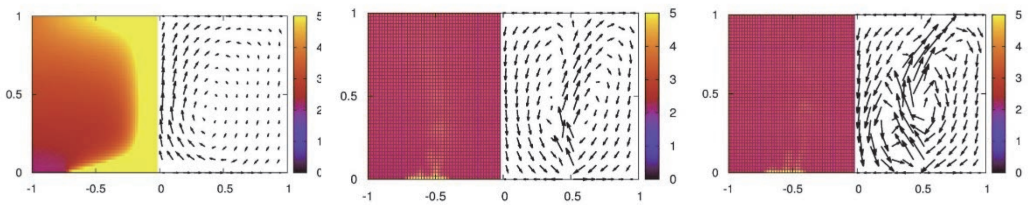
Da, where the volume average of the temperature T is defined as $<T>_V = \left(\frac{1}{V}\right)\int T\,dV$. It is found that the increase of the average temperature is independent of Da, which means that the rate of input heat intensity is independent of the configuration of the foodstuff in the surrounding liquid. On the other hand, Fig.9 (b) shows the standard deviations of the velocity (q=|v|) and temperature. It is found that the deviation of the velocity increases and the temperature decreases with the decrease of Da, which means that the temperature distribution becomes homogeneous due to larger velocity disturbances with the increase of Da.



(a) (b)

Figure 9 Variations of temperature and velocity with time when Da=10^{-5}, where the average temperature is denoted in (a) and the standard deviations of temperature and velocity in (b).

On the other hand, Fig. 10 shows the temperature and velocity distributions for different Da at a particular time , where (a) denotes for Da=10^{-5}, (b) Da=10^{-4} and (c) Da=10^{-3}. It is found from the figures that the increase of Da enhances the convection and, as a result of this, makes the temperature uniform over the container.

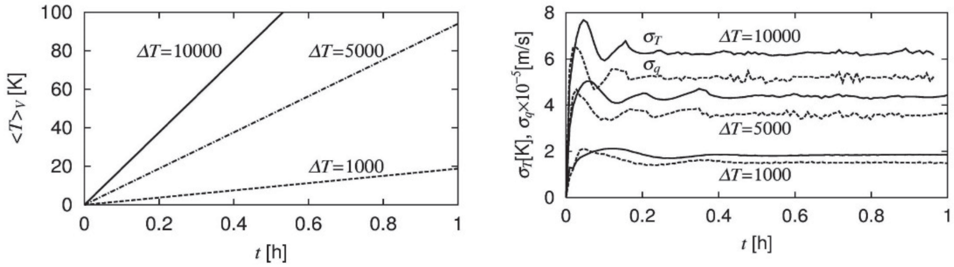


(a) (b) (c)

Figure 10 Variations of the temperature and velocity distribution with Da, where (a) Da=10^{-5}, (b) Da=10^{-4} and (c) Da=10^{-3}.

3.2.4 Larger heat intensity (2D analysis)

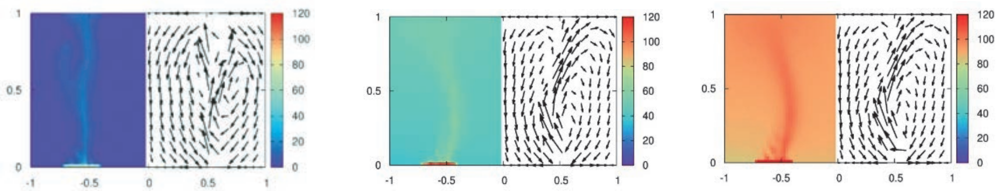
Next we consider the case when larger heat intensity of $\Delta T(=q_{in}\Delta S)$ when Da=10^{-5}. Figure 11 shows the



(a) (b)

Figure 11 Variations of temperature and velocity with time when Da=10^{-5}, where the average temperature in (a) and the standard deviations of temperature and velocity in (b).

temperature and velocity variations with heating time when ΔT=1000 ($q_{in}\Delta S$=54.2W), 5000 (271W) and 10000 (542W), where the average temperature is shown in (a) and the standard deviations of the velocity and temperature in (b). It is found that the increase rate of the average temperature is proportional to the input heat intensity ΔT, while the standard deviations of both velocity and temperature increase with the increase of ΔT as long as the values of Da are kept unchanged. This means that the larger heat intensity makes the temperature and velocity fields more quickly disturbed. On the other hand, Fig. 12 shows the time evolutions of temperature and velocity distributions for ΔT=5000, where (a) denotes for t= 0.027 [h], (b) for t=0.5[h] and (c) for 1[h]. It is found from these figures that the average temperature increases with time keeping overall temperature distribution to be homogeneous, which is seen from the transition of background color from blue to red with the increase of time.



(a) (b) (c)

Figure 12 Variations of the temperature and velocity distribution with heating time when ΔT=5000, where t= 0.027 [h] in (a), t=0.5[h] in (b) and 1[h] in (c).

On the other hand, rather strong velocity distributions are kept for all time, which lead to the larger convection and homogeneous temperature distribution. As a result, we can say that convection hardly depends upon the average temperature <T>v for a constant ΔT or input heat flux, but the Darcy number plays an important role on the convection.

We finally show in Fig. 13 the average Nusselt number Nu defined as $\mathrm{Nu} = \sigma \int (vT)\, dV$ over the volume of liquid when ΔT=100 (K), where $\sigma = (\rho c)_f / (\rho c)_m$ as is shown in section 2.2.1. When Da varies from 10^{-8} to 10^{-1}, we can see from the figure that Nu drastically decreases for smaller Da than 10^{-5}. Since Nu implies the ratio of convective to conductive heat transfer, we can see that the conductive heat transfer becomes dominant for smaller Nu, whereas the convective heat transfer becomes dominant for larger Nu which takes the maximum in the vicinity of Da=10^{-4}. For Da giving the maximum Nu, we can obtain the following relation between Dp/H and ϕ from the definition: $(Dp/H)^2\phi^3 / [180(1-\phi)^2] \sim 10^{-4}$. This relation means that the boiling process in the cooking is most effective in the sense that the heat transfer is most prompted. In fact, as is seen from Fig. 5 in the case of Large potatoes, the convective motion is dominant in the above region Da~10^{-4} and then the temperature deviations in the container are sufficiently small. Consequently, the above relation gives the optimum relation between the averaged size and the volume ratio of the potatoes, which leads to a homogeneous heating of the foodstuff in the container, and this may be useful for energy saving cooking.

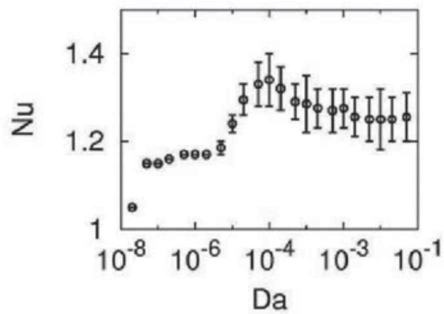


Figure 13 Variations of average Nu with Da.

4 Conclusions

We finally conclude our results as follows:

- The experiment and the lumped (one-dimensional) model analysis show that the temperatures of the liquid and potatoes increase almost at the same rate when Dp is small (smaller Da), while the temperature of the potatoes increases more slowly than the liquid when Dp is larger (larger Da).
- The convective motion is mainly determined by Da rather than the input heat flux ΔT, where temperature deviation becomes smaller and velocity deviation becomes larger for larger Da and vice versa.
- The optimum size Dp of the potatoes and the liquid volume ratio ϕ can be determined from $Dp^2\phi^3 / [180(1-\phi)^2H^2] \sim 10^{-4}$, which leads to the homogeneous heating of the foodstuff in the container.

Acknowledgment

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